values
education

# Values in mathematics

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## Part 1: A framework for values in mathematics and mathematics education

### Introduction

Values can be related to mathematics and mathematics education in, and through, three distinct but inter-related contexts, as indicated in Table 1:

**Table 1: contexts and related values**

| Context | Related values – the nine *Values for Australian Schooling* |
|---|---|
| Cultural and historical | • Doing Your Best – *seek to accomplish something worthy and admirable, try hard, pursue excellence*<br>• Respect – *treat others with consideration and regard, respect another person's point of view or perspective*<br>• Understanding tolerance and inclusion – *be aware of others and their cultures, accept diversity within a democratic society, being included and including others* |
| Socio-political | • Fair go – *pursue and protect the common good where all people are treated fairly for a just society*<br>• Honesty and Trustworthiness – *be honest, sincere and seek the truth*<br>• Integrity – *act in accordance with principles of moral and ethical conduct, ensure consistency between words and deeds* |
| Individual | • Care and compassion – *care for self and others*<br>• Responsibility – *be accountable for one's own actions, resolve differences in constructive non-violent and peaceful ways, contribute to society and to civic life, take care of the environment* |

To understand the nature of possible relationships between values and these three contexts, and their implications for implementation of the study of values within the mathematics curriculum, it is important to have a critical framework to inform consideration of contemporary and historical ideas and practices.

These considerations encompass questions about the nature of mathematics and the practices of mathematicians, the role of, and regard for, mathematics in various cultures and societies over time (including our own), and *how* and *why* decisions about *what* mathematics is made available to *whom*, by what *means*, and for what *purposes*, as part of school mathematics education. The latter is particularly important in the later adolescent stages of secondary education, where students undertaking study of a senior education certificate make choices in relation to their own interests (since at this level they can, in principle at least, exercise some choice) but also about possible educational pathways leading to future opportunities for further study, training or transition to work.

## Values and the philosophy of mathematics and mathematics education

There is a close relationship between philosophies of mathematics – that is attempts to explain the *nature* of mathematical objects and knowledge, *how* it arises, and *why* it is believed to be true, the nature of *mathematical inquiry*, and approaches to the teaching and learning of school mathematics. A key aspect of mathematics education is to investigate the nature of this relationship and other issues such as the real and perceived *applicability* of mathematics, the *aims and goals* of mathematics education and how these relate to what we know about *how students learn* mathematics and what are regarded as *effective teaching strategies*. This is an important starting point for teachers, as it provides an opportunity for them to clarify their own views on these matters, compare them with their own experiences, and the experiences of their colleagues, and be aware of the diversity of views held by mathematicians, philosophers, educators, fellow teachers, students and their parents on fundamental issues. Teachers can find useful and accessible discussion and background on these and related issues in Davis (1984), Ernest (1991), Hersch (1979), Jacquette (2002), Kline (1980) and Tout & Motteram (2005).

Two key philosophies of mathematics are *platonism* and *social constructivism*. In the former, mathematical objects and truths are conceived as having existence independently of the human mind, as part of the infrastructure of the universe, or Plato's world of ideals. Here mathematics is related to the 'harmony of the spheres' or alternatively, 'the mind of god'. Mathematical truths are held to be necessary by virtue of the way things are. Social constructivism argues that mathematics is a human endeavour, a very subtle, sophisticated, logical and powerful one, but ultimately it is a special human language, and its truths arise from the structure of this language, and are subject to the discourse of mathematical inquiry (see, for example, HREF1). Both of these philosophies of mathematics (and other philosophies of mathematics) have their adherents, depending on their individual and joint beliefs, values and preferences, and these in turn influence views on mathematics education.

The difference between these two philosophies is sometimes characterised by the choice between belief in whether mathematical truths are fundamentally *discovered* or *invented*. For late adolescent age students in the senior secondary years, the basis of the claims of any domain become increasingly of interest in their own right, especially where there is robust discussion and even controversy in the area (see, for example, HREF2). It is important for students to be aware that the certainty of mathematical knowledge, and any basis for such certainty, is contestable.

## Aims of school mathematics education

In *The Philosophy of Mathematics Education* (1991) Paul Ernest provides a framework for describing how different philosophies of mathematics are held by various social groups, which have educational aims for mathematics according to their corresponding beliefs, values and preferences (see also HREF3, HREF4). These aims may be referenced with respect to *who* makes decisions about *what* mathematics for *whom* and *how* this mathematics may be deployed. This process is often characterised in terms of different types of mathematical knowledge and capabilities, as outlined in Table 2 (Ernest, 2004):

**Table 2 Different types of mathematical knowledge and capabilities**

| Type of knowledge | Associated capabilities |
|---|---|
| Utilitarian knowledge | To be able to demonstrate useful mathematical and numeracy skills adequate for successful general employment and functioning in society |
| Practical, work related knowledge | To be able to solve practical problems with mathematics, especially industry and work-centred problems |
| Advanced specialist knowledge | To have an understanding and capabilities in advanced mathematics, with specialist knowledge beyond standard school mathematics (including advanced high school specialist study of mathematics to knowledge of university and research mathematics) |
| Appreciation of mathematics | To have an appreciation of mathematics as a discipline including its structure, sub-specialities, the history of mathematics and the role of mathematics in culture and society in general |
| Mathematical confidence | To be confident in one's personal knowledge of mathematics, to be able to see mathematical connections and solve mathematical problems, and to be able to acquire new knowledge and skills when needed |
| Social empowerment through mathematics | To be empowered through knowledge of mathematics as a highly numerate critical citizen, able to use this knowledge in social and political realms of activity. |

## Links to curricula in Australian states and territories

Since the publication of *A National Statement on Mathematics for Australian Schools* (1990), which described mathematical understandings, skills, knowledge and processes with respect to eight aspects: *Attitudes and appreciations*, *Mathematical inquiry*, *Choosing and using mathematics*, *Space*, *Number*, *Measurement*, *Chance and data* and *Algebra*, state and territory curriculum documents have incorporated curriculum design constructs related to what may be broadly termed 'working mathematically'. In some states and territories this is dealt with in compulsory years (typically to the end of Year 10) curriculum documents through and explicit curriculum organiser (eg a strand) of this (or similar) name, such as in NSW, ACT, Victoria, SA and WA. In other states and territories such as in QLD, Tasmania and NT it is described by way of an overarching approach or underpinning theme. In the 2005 Nationally Consistent Curriculum Outcomes (NCCO) project for Mathematics, *Working mathematically* was one of five curriculum organisers used to develop Statements of Learning (opportunities to learn) for junctures at Years 3, 5, 7 and 9. This aspect of curriculum provides an explicit and natural context for consideration of issues either relating to values, beliefs and preferences to mathematics, or involving mathematics as a form of rational inquiry applied to contexts where values, beliefs and preferences are keys aspects of a problems, task or situation. For example, global warming is an issue which elicits strong statements of values, beliefs and preferences held by various participants and stakeholders in private, public and global discussions. Mathematics, science and technology all have a key role to play in analysing data, developing models and making inferences and predictions, which can be used to critically analyse claims, carry out cost/benefit analyses (which necessarily involve values) and explore futures scenarios (where related beliefs and values are supported or otherwise by willingness to invest resources, modify behaviours and the like).

In the senior secondary mathematics curriculum there is a substantive focus on discipline content related to facilitating student pathways to further study (Paul Ernest's advanced specialist study in Table 1), and the relevant curriculum design is developed accordingly. Nevertheless, there is substantial scope for incorporation of values related material through:

- Investigation of the historical development of aspects of mathematics, and the lives of mathematicians thus involved, and the culture and society in which this work occurred
- Mathematical inquiry related to contemporary social, political, economic and other issues of interest, for example, quality and flow of water in the Murray–Darling river system and the impact of this on the livelihood of people; the allocation (and re-allocation) of resources through the taxation system; the role of gambling in Australian society

Consideration of how effectively numerate citizens are able to contribute to debate in the body politic and ensure that they are able to be fairly and justly treated in their various transactions with private and public businesses, companies, authorities, institutions and organisations.

## References

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## Web references

HREF1: http://www.cse.ucsd.edu/users/goguen/pps/real.pdf – *Reality and human values in mathematics*, Joseph Goguen, University of California. Site last visited 12/05/06.

HREF2: http://www.cs.auckland.ac.nz/CDMTCS/chaitin/lowell.html – *A Century of Controversy over the Foundations of Mathematics*, Greg Chaitin (1999). Site last visited 12/05/06.

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HREF3: http://www.people.ex.ac.uk/PErnest/pome18/PhoM_%20for_ICME_04.htm. – *What is the philosophy of mathematics education?* Paul Ernest, Philosophy of Mathematics Education Journal. Site last visited 12/05/06.

HREF4: http://www.aare.edu.au/99pap/bis99188.htm – *Values in Mathematics Education: Making Values Teaching Explicit in the Mathematics Classroom,* Alan Bishop, Gail FitzSimons & Wee Tiong Seah, AARE, 1998. Site last visited 12/05/06.

# Part 2: Values in mathematics – a history and culture perspective

## Introduction

The contemporary discipline of mathematics has been developed over millennia to incorporate a rich history and tradition that draws on the work of many societies and cultures. The aspects of mathematics developed, the way mathematics was regarded, and the approaches to working mathematically have also varied across these societies and cultures, and changed over time. In some societies and cultures, knowledge of mathematics and its applications has been the province of an elite class of philosophers, priests or scribes, in others merchants and explorers, in yet others scientists and industrialists. In contemporary Australian society, strong numeracy is expected of all adolescents, and a sound mathematical background is an essential requirement for many pathways of further study. Mathematics is a required part of the compulsory years curriculum, and a key enabling study, and also in its own right, for senior secondary students in the late adolescent years.

*Computation* and *proof* are two complementary aspects of mathematics that are often valued differently in relation to the types of knowledge and associated capabilities described in Table 2 of Part 1 and related aims for school mathematics. Throughout history various technologies, from abacuses and counting boards, to pen-and paper algorithms, drawing tools, tables, slide rules, mechanical, electrical and electronic devices have been used to facilitate efficient computation (numerical, graphical and symbolic). Indeed, the Latin word *calculus* (from which we obtain the word *calcul*ation) means 'a small stone or counter for reckoning' of the kind used as a counter on abacuses or counting-boards around the world well into the 16th century AD. The use of various forms of technology in school mathematics, with respect to the aims of school mathematics, student cohort, and purpose is an area of mathematics education subject to robust debate based on the beliefs, values and preferences of different social groups. In particular, the issue of what students should know and be able to do with, and without, the assistance of technology is an area of keen interest and discussion. Australia is a highly technological society - state and territory curricula preclude, permit, or expect (with different sets of underpinning values and related expectations) students to use technologies in their mathematical work and corresponding assessments - depending on the purpose, nature and scope of the course being studied.

The following contexts and activities are designed to assist students to develop a historical perspective on the use of technology in mathematics and its applications in different societies and cultures, and explore social and cultural values related to this use in these contexts. This work connects with the value:

- Understanding tolerance and inclusion - be aware of others and their cultures, accept diversity within a democratic society, being included and including others

They can then proceed to critically reflect on the role of technology in mathematics and its applications in their own culture and society (in and out of school) and discuss related beliefs, values and preferences. This work connects with the values:

- Doing Your Best - seek to accomplish something worthy and admirable, try hard, pursue excellence
- Respect - treat others with consideration and regard, respect another person's point of view or perspective

By showing how various people tried to tackle challenging problems and issues of their time, and also how people from the Middle-East, Europe, India and China have all made contributions - often shared through exploration, trade and commerce - to the development of mathematics, these three values can be explored and affirmed.

In each case it will be important to relate the mathematics in the historical situation and the technology utilised, to the formulation, solution and interpretation of related problems in the contemporary curriculum. This can be done, either as a one-off activity, a collection of activities around a topic, or as a general approach deployed as applicable to engage student interest, by:

- the *teacher* in terms of providing context for the study of a particular aspect of mathematics and related problems from historical times (eg the development and use of geometry and mensuration in ancient agricultural societies ~ 2 000 BC, such as Egypt and Babylon);
- *students* carrying out a short and focused historical research project into a given period (the development of general purpose computers and discrete mathematics to solve optimisation problems for business and industry in the mid 20th century AD);
- *teacher and student* investigation of the historical development of an idea over an extended period, for example, the solution of linear, quadratic and cubic equations in one variable, and related numerical, graphical and analytical methods (the Ancient orient, Baghdad ~ 800 AD, China in the 14th century AD, Italy in the 16th century AD and Europe in the 19th century AD); and
- exploration of different technologies used for mathematics in Australian schools over recent generations from the 1960's to the present time.

There will be greater flexibility in the penultimate year of senior secondary mathematics (typically Year 11) to incorporate a sequence of several lessons with a historical basis for the development of a topic with related materials, or perhaps a historical investigation in its own right. In the final year of senior secondary mathematics (typically Year 12) it is more likely that teachers will opt to avail themselves of such material to introduce a topic, or focus on a specific concept or technique.

Some useful references are *A Concise History of Mathematics* (Struick, 1948); *Exploring Mathematics through History* (Eagle, 1995) – this reference includes lesson descriptions and student activities; and *The Crest of the Peacock – Non-European Roots of Mathematics* (Joseph, 1995) – this book highlights the contribution of different cultures to mathematics.

A web search using "history of mathematics" as the search term will produce many relevant sites, which are often cross-referenced. See also the following websites:

- http://www.math.sfu.ca/histmath/;
- http://www-gap.dcs.st-and.ac.uk/~history/;
- http://archives.math.utk.edu/topics/history.html; and
- http://www.maths.tcd.ie/pub/HistMath/Links/.

A web search using "history of calculating devices" or "history of calculators" or "history of computers" as the search term will produce many relevant sites, which are also often cross-referenced (for further detail it is also useful to specify *mechanical*, *electrical* or *electronic*). For material related to the historical development of various *technologies* for computation see:

- http://en.wikipedia.org/wiki/History_of_computing; http://www.diycalculator.com/cool.shtml#Hist;
- http://www.xnumber.com/xnumber/mechanical1.htm; and
- http://www.vintagecalculators.com/html/calculator_time-line.html

The following discussion outlines several illustrative contexts. An important consideration in the use of historical materials is *anachronism*, or the *interpretation* of such materials in the light of contemporary ideas and approaches that ascribes an understanding or meaning *beyond that* of the originals. For example, while the Egyptians did solve elementary linear equations in a single variable, to say that they solved linear equations such as $3x + 11 = 23$ is anachronistic. The notation that used such symbols as + and =, Hindu-Arabic numerals, and variables such as *x* was not fully realised until the $17^{th}$ century in Europe and England, and even then alternative notations continued to be employed. The Egyptians would certainly not have *expressed* equations in this form. Thus, wherever possible, it is important to use primary source materials, or close translations of these materials, as well as secondary source materials and commentary.

## Some possible questions

To assist in discussing values and mathematics in these contexts, the following questions could be used as prompts:

- What was the cultural context, what was society like at that time, and what were the important beliefs and values of that society and culture?
- What mathematics was developed, by whom, and for what purposes?
- What sort of mathematics was used by the various social classes in that culture and society?
- What views were held on the roles of computation and proof? What technologies were employed, and by whom?
- What kind of education did people receive? How does this compare with our society?
- How were people from different social and cultural groups regarded and treated?

## Contexts

### Egypt and Babylon 2 000 – 3 000 BC

Egyptian and Babylonian civilizations, originally based on an intensive form of agriculture associated with rivers and irrigation, were amongst the first to evolve more advanced forms of mathematics from earlier Neolithic communities in their region. The state had a strong association with religion and the priest class were often central to the bureaucracy and the use of mathematics. An initial emphasis on practical applications in arithmetic and geometry gradually evolved into more complex forms. Three areas of interest, in which comparative study could be made, are:

- the use of different *mensuration* techniques and formulas for lengths, areas and volumes and related practical problems
- the forms of arithmetic calculation, for example the Egyptian techniques for calculation with *unit fractions* using tables (technology), and the Sumerian/Babylonian *sexagesimal* place value system with its implicit use of zero

- the elementary *linear* (*aha* = *heap*) algebraic techniques of the Egyptians for solving certain practical problems, and the more sophisticated techniques of the Babylonians, for solving *quadratic* and some *cubic* and *exponential* equations using tables (technology) and linear interpolation.

Some useful websites are:
http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fractions/egyptian.html;

- http://www-gap.dcs.stand.ac.uk/~history/HistTopics/Babylonian_and_Egyptian.html;
- http://en.wikipedia.org/wiki/Egyptian_mathematics; and
- http://en.wikipedia.org/wiki/Babylonian_mathematics .

**Baghdad 9th Century AD**

Muhammad ibn Musa al-Khwarizmi wrote the first major text (*Hisab al-jabr wal-muqabala*) on what we call *algebra* in Baghdad in around 825 BC, at the request of the then Imam to produce a text on calculating what is easy and practical (what we might call 'applicable maths'). Instead he wrote an algebra text, focusing on the solution of linear and quadratic equations, which while it contains some practical examples, is really about algebraic reasoning. Arabic scholars at the time had access to works from both Greek and Indian (Hindu) traditions of mathematics, and played a key role in the introduction of the Hindu-Arabic numeration system to the west. Their reasoning was often justified by appeal to geometric arguments (technology - diagrams) following in the Greek tradition, and used in algebra experience material (AEM) or 'algebra blocks' today. Thus these scholars integrated the Oriental emphasis on computation with the Greek emphasis on proof. Our word *algebra* comes from one of the processes al-Khwarizmi used *al-jabr* or 'restoring' (transposing a quantity from one side of an equation to the other). It is from the Latinized version of his name (*al–Gorisme* to *algorisme*) that we get the word *algorithm* for a computational process. Some aspects of Arabic mathematics that might be investigated are:

- al-Khwarizmi's problems and their methods of solution
- Omar Khayyam's work (12th century AD) on solving cubic equations geometrically by considering then intersection of conic sections (technology - geometric constructions) Khayyam, the author of the *Rubaiyat,* is also well known as a Persian philosopher and a poet. Many of his poems can be read as a critical socio-political commentary of the times.
- The role of Arab scholars in the transmission of the Hindu-Arabic number system to the West, and the evolution of our number system, including zero.

Some useful websites are:

- http://www-history.mcs.st-andrews.ac.uk/HistTopics/Arabic_mathematics.html;
- http://en.wikipedia.org/wiki/Islamic_mathematics;
- http://www.unc.edu/~rowlett/units/roman.html;
- http://www.scit.wlv.ac.uk/university/scit/modules/mm2217/han.htm
- http://www.geocities.com/rmlyra/arabic.html

**China 14th Century AD**

Chinese mathematics developed in an essentially unbroken tradition over several millennia, and evolved a sophisticated computational mathematics (technology, bamboo rods, abacus and counting board) with algebraic problems solved numerically by manipulation of coefficients. From as early as the second millennium BC the Chinese had a regular decimal place value system, and, like the Babylonians, used a blank to designate zero. Some aspects of Chinese mathematics that might be investigated are:

- the use of the abacus (technology) for arithmetic calculation
- the use of the counting board (technology) to solve systems of simultaneous linear equations (by matrix transformations, including zero and negative elements)
- the use of the counting board to solve polynomial equations by numerical methods
- the Chinese approach to the Pythagorean relationship (theorem)

Some useful websites are:

- http://www-gap.dcs.st-and.ac.uk/~history/HistTopics/Chinese_overview.html
- HREF6: http://en.wikipedia.org/wiki/Chinese_mathematics
- HREF7: http://school.discovery.com/lessonplans/programs/advancedAlgebra/ (this website provides a detailed lesson sequence leading up to solving systems of simultaneous linear equations using Chinese methods from the 14$^{th}$ century AD);
- http://en.wikipedia.org/wiki/Abacus; and
- http://aleph0.clarku.edu/~djoyce/ma105/chinalg.html

**Europe 12$^{th}$ Century AD – 17$^{th}$ Century AD**

In the twelfth and thirteenth centuries AD, the Italian sea-faring cities of Genoa, Pisa, Venice and Milan carried out extensive trade with both the Arabic world and the Northern European countries. Leonardo of Pisa, who is also the Fibonacci (son of Bonaccio) of the breeding rabbit problem (the Fibonacci sequence) fame, travelled extensively in the Orient. In 1202 he wrote his *Liber Abaci* (book of the counting board) in which he describes various aspects of arithmetic and algebra, and, of course, his famous sequence. The *Liber Abaci* is one of the books by which the Hindu-Arabic numeration system was introduced into the West. This, and the related pen-and-paper methods of calculation were a radical and controversial new technology which replaced counting boards and abacuses over a period of some time and came into more common use during the 14$^{th}$ century AD (see: http://www-history.mcs.st-andrews.ac.uk/Biographies/Fibonacci.html )

In the 16th Century AD, the arithmetic of Robert Recorde is the first work written in English for the use of pen-and-paper technology for computation and hence more widely available to the general population than other scholarly mathematical works in Latin. Recorde introduced the use of some of the modern symbols for arithmetic and algebra, for example, the use of the symbol '=' for equality was introduced in his book the Whetstone of Witte (1557). He also solves linear and quadratic equations. The work of Recorde and other English and European mathematicians followed on from the introduction and dissemination of the Hindu-Arabic numeration system into Western Europe during the previous few centuries. Recorde's work was central to pencil and paper algorithms for calculation of arithmetic computations displacing counting-boards (see: http://www-history.mcs.st-andrews.ac.uk/Biographies/Recorde.html )

Logarithms were invented by John Napier in 1614 and later improved by Henry Briggs in consultation with Napier to produce logarithms to base 10 for general calculation and the first logarithm based straight slide rule developed by William Oughtred in 1622. Following improvements in arithmetic algorithms during the 15th and 16th centuries, hand operated devices such as Napier's bones (which can be considered as a type of hand operated generalisation of the gelosia or lattice method for multiplication) and the slide rule where the user must actively manipulate the device at each stage of the computation process, are introduced to facilitate arithmetic computations. In conjunction with the invention of logarithms, there is the evolution of a sufficiently sophisticated mathematics to support the subsequent development of the early mechanical calculators (see: http://www-history.mcs.st-andrews.ac.uk/Biographies/Napier.html; http://www-history.mcs.st-andrews.ac.uk/Biographies/Oughtred.html )

The first recorded design for a mechanical calculating device has been attributed to Leonardo da Vinci (circa 1500) following the recent discovery of some of his notes in the National Museum of Spain that include a description of such a machine. While working models of da Vinci's machine have been made based on these notes in recent times there is no evidence that this idea went 'beyond the drawing board' in his own time. The first mechanical arithmetic calculating device, sometimes referred to as 'the calculating clock', was designed and constructed by Wilhelm Schickard in 1623 and was based on Naperian logarithms etched on rotating cylinders. It could carry out all four arithmetic operations and although the historical records indicate that working versions were manufactured none have survived to the present time. The first mechanical calculator produced during this time, of which an original model survives today, is Blaise Pascal's 'Pascaline', constructed in 1640. Independently, in 1673, Gottfried von Leibniz improved on Pascal's design with his 'Stepped Reckoner', which could compute all four arithmetic operations and was technically more robust than previous devices. Leibniz commented that '*…it is unworthy of excellent men to lose hours like slaves in the labour of calculation, which could be safely relegated to anyone else if machines were used.*" (see: http://www-history.mcs.st-andrews.ac.uk/Biographies/Pascal.html ; http://www-history.mcs.st-andrews.ac.uk/Biographies/Leibniz.html )

**Other contexts**

There are a range of other contexts that may be drawn on including:

- Charles Babbage's differential and analytical engines in the 19th Century
- The emergence of electrical calculators and computers in the early 20th century
- The development of thermionic, and subsequently semi-conductor calculators and computers since the mid 1950's
- The development of computer algebra systems (CAS) since the 1960s and the broader availability of hand-held scientific, programmable, graphics and CAS calculators since the 1970s

The use of 'modern' technologies, which have, in various combinations, powerful numerical, graphical and symbolic capabilities raises interesting *belief and value related* issues, contentions and questions with respect to *school* mathematics. In particular, these are linked to expectations for student capability with *mental*, *written* and *technology-assisted* approaches to mathematical skills and techniques such as calculation, drawing graphs and solving equations. There is interesting scope for inter-generational discussion; student's whose parents were at school in the 1960s and 1970s are likely to have used slide rules, log books (tables) or perhaps a mechanical calculator, as the most advanced technology available to them. In the 1980s electronic arithmetic and scientific calculators as well as early desktop computers played this role, while in the 1990s graphics calculators and spreadsheets (incorporating powerful statistical functions) were common advanced technologies for computation. In the 2000s this role is most likely to be played by hand-held or software based computer algebra systems (CAS). While abacuses and counting boards are not part of the technology repertoire of the curriculum in Australian states and territories (but continue to be in several Asian countries) 'by-hand' pen-and-paper technology is still a central aspect of the senior secondary mathematics curriculum.

### A possible approach to assessment

The following is a suggested approach to assessment via a short report on research directly related to a particular aspect of mathematics in the curriculum, in this case based on the context of *solving equations* in work on *function* and *algebra*. This would also involve consideration of the various methods of calculation used. For example, when there is a formula or rule (broadly interpreted) relating a dependent variable to one or more independent variables, knowing sufficient information about some of these variables and seeking to determine the value of the other variable involves both *direct* and *inverse* calculations.

The teacher, in an introductory lesson, could model the required research approach in a given context. For example, in discussing linear equations, the teacher might develop the approach used by Robert Recorde and:

- describe the nature of the society or culture (including their beliefs and values) which used this mathematics, for what purposes, and who utilized it
- outline how the problem was formulated, solved (including the use of technology – tables and/or other devices) and interpreted in context
- (have students) solve several problems of a like kind using this methodology
- carry out the corresponding analysis in modern terms, and discuss any special features and/or limitations of the historical approach, any insights that may be gained from consideration of the approach(es) used in the historical context
- link the three previous points to values in mathematics in the historical context and discuss how they might relate broadly to the set of Australian values

The teacher would also need to discuss *how* they obtained the background historical material needed to prepare for, and develop, the lesson they had presented as part of the modelling process (unless something like this has been done at an earlier stage).

Students could subsequently work over several lessons (2 or 3) individually or in pairs, to prepare their report (using a structure and format specified by the teacher) on one of a selection of topics related to the *solving equations* area of study. A set of around 10 – 15 topics such as those listed earlier, or other topics such as *Diophantine equations* or *numerical methods*, such as Horner's rule, could be used depending on how closely linked the work is required to be to the development of specific content. The report could be assessed for level of performance via a rubric or criteria based on the five aspects outlined above and using a 3, 4 or 5 point scale as appropriate. In the final year of senior secondary study, such an approach, if used, would need to be consistent with the requirements and expectations for school based assessment in the relevant jurisdiction. The following website provides guidance on assessment: http://pareonline.net/getvn.asp?v=7&n=25 (discusses rubrics and their uses).

**References**

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